Journal of Sound and Vibration 330 (2011) 868–882

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Shock isolation using an isolator with switchable stiffness

D.F. Ledezma-Ramirez *, N.S. Ferguson, M.J. Brennan

Institute of Sound and Vibration Research, University of Southampton, Southampton SO17 1BJ, UK

ARTICLE INFO

Article history:
Received 12 August 2009
Received in revised form
14 September 2010
Accepted 16 September 2010
Handling Editor: J. Lam
Available online 16 October 2010

ABSTRACT

A semi-active control strategy is presented for the shock isolation of resiliently mounted equipment where the isolator has light damping. This is achieved by switching the stiffness of the isolator between a high-state and a low-state. The control strategy involves two stages: the first stage involves the displacement control of the equipment during the shock, and the second stage involves suppression of the subsequent residual vibrations. The performance of the switchable isolation system is illustrated using a base-excited single degree-of-freedom system. It is characterized in terms of the maximum absolute acceleration and displacement of the isolated mass, the relative displacement between the base and the mass, and the effective damping ratio of the system. Provided that the damping in the isolator is light, it is found that the semi-active system can outperform a linear passive system during both stages of control.

1. Introduction

Shock is a sudden excitation normally of short duration, which can be highly detrimental because it typically involves high forces, displacements or stresses [1]. For sensitive equipment, excessive shock response may cause damage, because the allowable levels of stress or strain resulting from the displacement, velocity, or acceleration may be exceeded. Moreover, if the equipment is resiliently mounted and positioned in a finite space, a large relative displacement between the equipment and the host structure, as a result of the shock, could cause damage as the equipment could impact on an adjacent structure. Passive shock isolators, which also act as vibration isolators, are often used. They have many different configurations based on rubber pads, helical springs, composite isolators using metallic and viscoelastic elements, and other similar devices [2].

The analysis of shock isolation is often performed using a single degree-of-freedom (sdof) model with linear elements subjected to different pulse or step functions [3]. Linear models of the isolator can provide estimates of the response, but nonlinear models may be required to give an accurate prediction. Isolator models, which have hardening or softening behaviour, are frequently used [4,5]. For shock isolation, a lightly damped isolator with a softening characteristic can outperform a linear isolator.

Optimum shock isolation involves a particular performance index, for example, the absolute acceleration of the equipment, and a design constraint such as the peak relative displacement between the equipment and the host structure [6]. With passive isolators there is an inevitable compromise between the performance index and the design constraint. Semi-active shock isolation offers a way of overcoming this compromise. In such systems, advanced knowledge of the shock input can also lead to a substantial improvement in performance [7].

In the area of semi-active control, several switchable and variable stiffness strategies have been investigated. Winthrop et al. [8] reviewed the most notable variable stiffness related studies, and also presented a method for the selection of a

* Correspondence to: Permanent address: Facultad de Ingeniería Mecánica y Eléctrica, Universidad Autónoma de Nuevo León, Av. Universidad s/n, San Nicolás de los Garza, Nuevo León, México 66451, Mexico. Tel.: +52 81 83294020x5762; fax: +52 81 83320904.
E-mail address: diego.ledezmard@uanl.edu.mx (D.F. Ledezma-Ramirez).

0022-460X/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jsv.2010.09.016

variable stiffness device. One particular control strategy for transient vibration suppression utilises on–off logic to connect/disconnect a stiffness element. A strategy was originally proposed by Onoda and Minesugi [9] and Onoda et al. [10,11], based on variable structure theory, although Chen [12] had shown previously that it is possible to control structural vibration by varying stiffness, and that it is particularly useful in reducing the vibration of lightly damped systems. More recently, Jabbari and Bobrow [13] and Leavitt et al. [14] have considered the variable stiffness approach and also a resetting technique for a pneumatic valve based device. The resetting technique aims to extract energy at any time from a dynamic system and is equivalent to a high stiffness system at all times, but effectively resets the unstretched length of the equivalent spring element. This alternative approach can be considered to always provide a high stiffness, and if required energy can be dissipated more frequently and faster vibration suppression has been proposed [16]. The optimal performance for free vibration of such devices [14], incorporating a gas filled actuator, was found to be when the stiffness was reduced. The energy was dissipated when the velocity of the simple single degree-of-freedom system had a change of sign. The resetting method, in principle, instantaneously restores the stiffness to the system, but if there is a delay in resetting the stiffness until the next crossing point through the equilibrium position then the resetting approach is identical to the variable stiffness approach. In the variable stiffness approach the low stiffness is present until the next crossing point through the equilibrium, so the energy stored in the stiffness is not as much as it is when the stiffness has been reset to its high value. Therefore, under excitation the resetting approach, which removes energy when it is at its peak stored values, performs better than a variable stiffness.

Complementary studies of variable stiffness devices are in the field of tunable vibration isolators. These possess high static stiffness and low dynamic stiffness. The mechanisms could be designed passively using the geometry so that they exhibit the low stiffness, or tangent stiffness, about the static equilibrium point. Alternatively, adaptive stiffness which can be added or removed, such as having an electromagnetic spring working in parallel with a mechanical spring [15] can be used to produce a tunable isolator. The recent work by Zhou and Liu [15] had the additional complication that both springs possess nonlinear characteristics, and the response under harmonic excitation displayed the jump phenomenon typical of hardening and softening nonlinearity. No shock results were investigated or reported.

This paper considers and applies the switchable stiffness strategies described by Onoda and Minesugi [9] and Onoda et al. [10,11] to the problem of shock isolation. The problem involves two phases: the first is the suppression of the response during the shock, and the second is the suppression of the subsequent residual vibrations. Concerning the latter phase, switching stiffness strategies have been used by Onoda et al. to suppress free vibration, but the analysis is extended in this paper to investigate the way in which energy is dissipated in such a system. To illustrate the control strategies a sdof system is used as a model problem.

2. Typical response of passive shock isolation systems

In this paper, the performance of the switchable stiffness semi-active control strategy is compared with that of passive shock isolation, so the performance of a passive system is briefly reviewed first. The maximax displacement response for a sdof system subject to shock is mainly controlled by the ratio of the duration τ, of the shock pulse, and the natural period T, of the system [1]. This can be seen when examining the shock response spectra (SRS) [1], an example of which is shown in Fig. 1, for a base-excited sdof system; in this case, the displacement input is a versed sine. In Fig. 1, v_m is the maximum absolute displacement response of the isolated mass and ξ_m is the maximum displacement of the shock pulse. Three regions are depicted: the isolation region, the amplification region, and the quasi-static region. Typical normalised time responses for these regions are shown in Fig. 2. It can be seen that the maximum response of the system in the isolation zone, τ/T=0.25, is smaller than the amplitude of the input; however in the amplification region, τ/T=1, the maximum response is larger than the amplitude of the input. In the quasi-static zone, τ/T=4, the response closely follows the input, but with small amplitude residual vibrations. If the shape and duration of the input cannot be modified, the natural period should be chosen appropriately to achieve isolation; an ideal shock mount should be as soft as possible so that the ratio $\tau/T \leq 1$. In practice, there is often a limitation on the minimum allowable stiffness. The main effect of any damping present is to suppress the residual vibrations.

3. Control of stiffness during the shock

The undamped sdof system with switchable stiffness considered in this paper is shown in Fig. 3. It comprises a mass m, supported by two springs in parallel, k–Δk, which is termed the primary stiffness, and Δk, which is termed the secondary stiffness. The orientation of the system is horizontal to avoid the effect of gravity which can lead to an undesirable change in equilibrium position as the stiffness is switched [16], thus the system is side excited by a versed sine displacement shock excitation $\xi(t)$ applied in the support and given by

$$\xi(t) = \left\{ \begin{array}{ll} \dfrac{\xi_m}{2}\left(1-\cos\left(\dfrac{2\pi t}{\tau}\right)\right), & 0 \leq t \leq \tau \\ 0, & \tau < t \end{array} \right\} \tag{1}$$

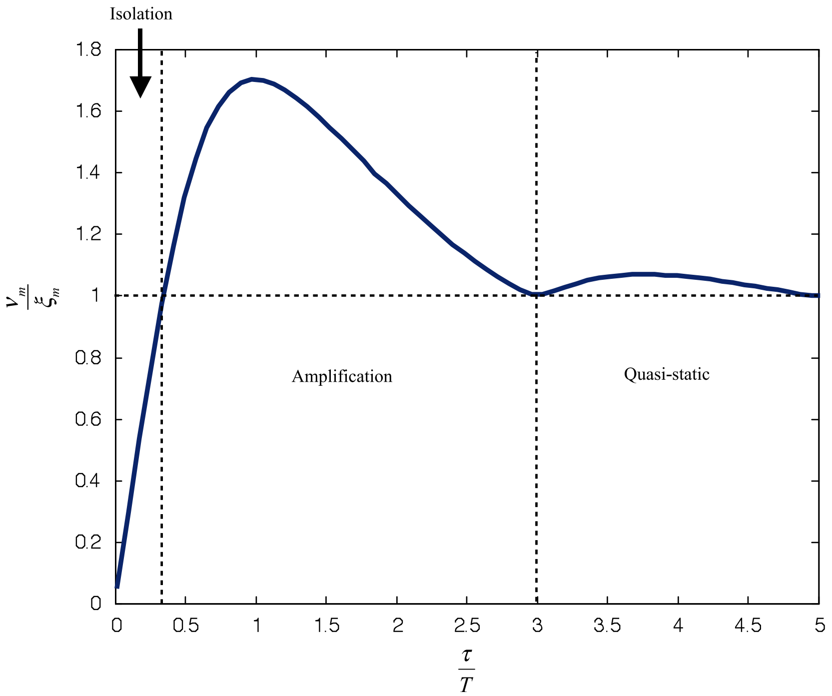

Fig. 1. Example of a shock response spectra (SRS) for an undamped sdof system, excited at the base by a versed sine pulse. Three typical regions are shown, namely the isolation, amplification and quasi-static regions. The axes are the normalised displacement response (maximum response divided by maximum input displacement) and the normalised pulse duration (pulse duration divided by the natural period of the system).

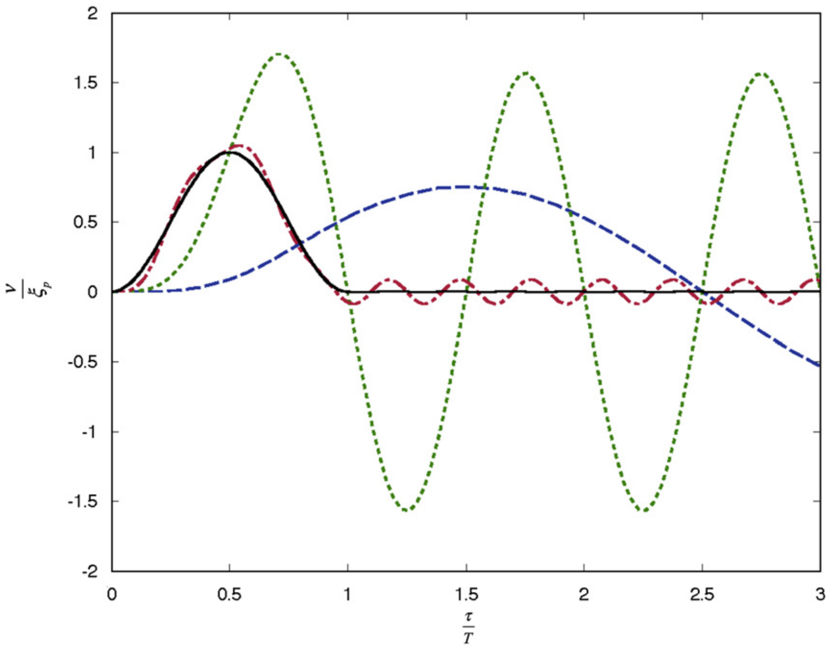

Fig. 2. Typical normalised displacement time responses for the base excited system with the SRS shown in Fig. 1. (——shock pulse input; — isolation region behaviour corresponding to $\tau/T=0.25$; · …amplification region behaviour for $\tau/T=1$; – · – quasi-static region behaviour for $\tau/T=4$).

The versed sine excitation was chosen as it results in no acceleration discontinuity on the isolated mass, and when the stiffness change is imposed at the time of application there is no acceleration discontinuity. For the duration of the shock the stiffness Δk is disconnected providing a softer support so that the maximum response is reduced. The equation of

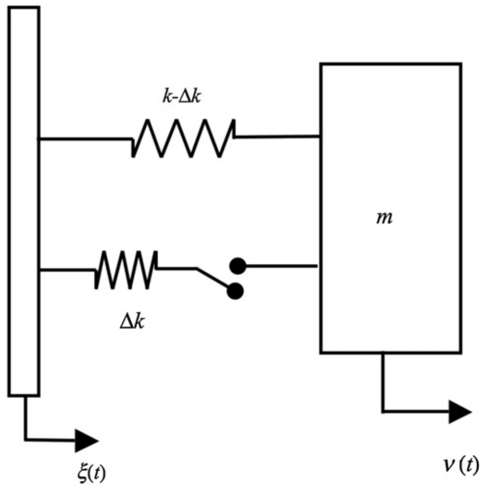

Fig. 3. Schematic of a sdof system with switchable stiffness under shock excitation $\xi(t)$ applied to the base.

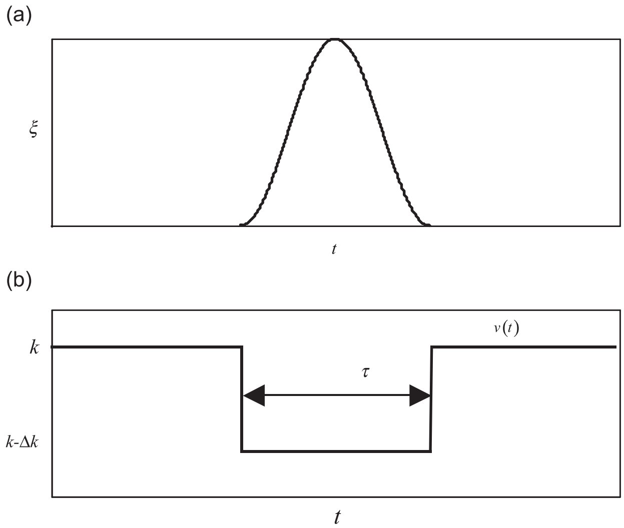

Fig. 4. Schematic to show the variation of the stiffness as a function of time: (a) versed sine shock input and (b) stiffness variation $k_{\text{efffective}}$.

motion of the system is

$$m\ddot{v}+k_{\text{effective}}v=k_{\text{effective}}\xi(t) \tag{2}$$

where $k_{\text{effective}}$ is the system stiffness, which can be either k or $k-\Delta k$ depending upon whether the secondary stiffness is connected or disconnected, respectively, as shown in Fig. 4. The change in stiffness is considered to be instantaneous for simplicity, but it is acknowledged that in practice it will take some time to achieve this because of sensor, controller and actuator delays. The effect of any delay can be considered for practical and stability considerations [16]. The total delay can be reduced, however, if there is advance information of the shock [7]. Assuming that the system is initially at rest with stiffness k, and that the shock occurs at t=0, the equations of motion for the system are

$$\begin{aligned} &m\ddot{v}+kv=0, && t=0, \quad t>\tau \\ &m\ddot{v}+(k-\Delta k)v=(k-\Delta k)\xi(t), && 0<t\le\tau \end{aligned} \tag{3}$$

This system of piecewise linear equations of motion is solved numerically using the Runge–Kutta method, although analytical solutions are possible [16]. It is assumed that the stiffness change can occur such that there is no discontinuity in the displacement. The two instantaneous natural periods of the system corresponding to the high and low stiffness states are given, respectively, by

$$T_1=\frac{2\pi}{\omega_1}, \quad T_2=\frac{2\pi}{\omega_2} \tag{4a,b}$$

where

$$\omega_1 = \sqrt{\frac{k}{m}} \quad \text{and} \quad \omega_2 = \sqrt{\frac{k}{m}\left(1 - \frac{\Delta k}{k}\right)}$$

Changing stiffness shifts the response of the system to a different region in the SRS. Initially the ratio of the pulse duration to the natural period for the system is τ/T_1, which will determine the maximum response. However, when the stiffness is changed, the system has a new ratio τ/T_2. The objective is to change the stiffness of the system so that it behaves as if it is in the isolation region of the SRS. To evaluate the performance of the control strategy, three normalised response parameters are considered. They are the absolute displacement, $v_m/\xi p$, the relative displacement $(v-\xi)_m/\xi_p$ and the absolute acceleration $\ddot{v}_m/\ddot{\xi}_p$, where the subscript p denotes the peak amplitude value corresponding to the shock input, either displacement or acceleration and the subscript m defines the maximax response. Ideally, a shock mount should minimize all of these parameters; however, there is a compromise between them [1]. A softer mount will reduce the maximum displacement and acceleration of the supported mass, but at the cost of increasing the relative displacement, which means more space is required. Moreover, if the mount comprises helical springs, there is a risk of the coils making contact thus forming a rigid link. As this switchable stiffness strategy is based upon a stiffness reduction, the relative displacement will probably increase, but the absolute responses are expected to decrease. Hence, even though the reduction of the stiffness will probably cause a larger relative displacement or deflection for certain situations, this strategy has been chosen because it is expected to reduce absolute responses. Nonetheless, the performance of the system is evaluated using all three responses. As a result, the primary parameters that have been considered for control are the maximum absolute displacement and acceleration. The maximum absolute acceleration could be indicative of damage, failure or human discomfort.

The overall behaviour of the control strategy is illustrated by some example time histories as shown in Fig. 5, for the absolute displacement and acceleration. Time histories are presented for representative values of the period ratio corresponding to the high-stiffness passive state, for τ/T_1=0.25 (isolation region), τ/T_1=1 and τ/T_1=2 (amplification region and quasi-static region, respectively) and for the system with a stiffness reduction of 50%. The effect of the stiffness change can be easily seen in Figs. 5(d) and (f), where a change in the oscillating frequency is noticeable. It is also noticeable how the change in stiffness causes the maximum response to decrease for some situations, especially when the shock pulse is short compared to the natural period of the system. As the pulse duration increases, however, the effect of the switching strategy can be detrimental, as seen in Figs. 5(c) and (f). For this particular value of τ/T_1=2 the passive system (dashed line) would have no residual response [1] but the change in natural frequency resulting from the stiffness switching changes the effective ratio between the pulse duration and the natural period causing the response to increase. To fully appreciate the effect in the response, it is necessary to obtain an index of performance using the passive response as a reference, for the several parameters involved. To illustrate this effect of stiffness reduction on the system response, the ratio between the absolute maximum responses for the semi-active case and for the high-stiffness (passive) state for different parameter results are presented. These results are plotted as a function of the stiffness ratio $\Delta k/k$. If the response ratio is greater than unity then the response of the semi-active system exceeds the response of the original passive system, otherwise there is an improvement in the performance. The results are shown in Fig. 6.

Regarding the absolute displacement, shown in Fig. 6(a), the maximax response is effectively reduced depending upon the degree of stiffness reduction for short duration pulses compared to the natural period ($\tau/T_1 < 1$). Higher stiffness reduction results in a smaller displacement response. For example, there is a decrease of approximately 40% in the absolute displacement response for a stiffness reduction of 50%, i.e. $\Delta k/k$=0.5, and an initial period ratio τ/T_1=0.25. However, as the period ratio increases the reduction in the response decreases and higher stiffness reductions are necessary in order to obtain any significant advantage. Furthermore, the maximax response can be amplified if the initial period ratio is large. i.e. $\tau/T_1 > 1$ An example of this can be seen in Fig. 6(a) when τ/T_1=2. In this case, the response increases except when the stiffness reduction is very high, i.e. more than 90%. Effectively a greater reduction in stiffness is required to shift the system to the SRS isolation region, and if the stiffness reduction is not large enough the system may be shifted to the amplification region. This effect manifests itself when the period ratio is greater than 1, where a small increase in the system displacement occurs.

For the relative displacement, which is calculated from the difference in the base motion and the isolated mass displacement, the absolute value is plotted in Fig. 6(b) and the resulting behaviour is different. It has been normalised by the maximum absolute value of the relative displacement for the passive system, so one can see changes in the relative displacement rather than normalisation by the base input magnitude. In a passive system a softer support will result in a larger relative displacement. This improves the shock isolation since the elastic support is capable of storing more strain energy, which could be dissipated later via a damping mechanism. For a short duration input, e.g. τ/T_1=0.25, the relative displacement is very similar to that of the passive system regardless of the stiffness reduction. There is some benefit obtained for larger values of τ/T_1 between 0.5 and 1. Moreover, the relative displacement increases using the switched lower stiffness when the period ratio is higher than 2. This would be unacceptable if the relative displacement is restricted for an isolated mass inside a fixed enclosure.

The remaining response parameter of interest is the normalised absolute acceleration shown in Fig. 6(c). Broadly the trends are similar to those for the absolute displacement. High stiffness reductions generally lead to a decrease in

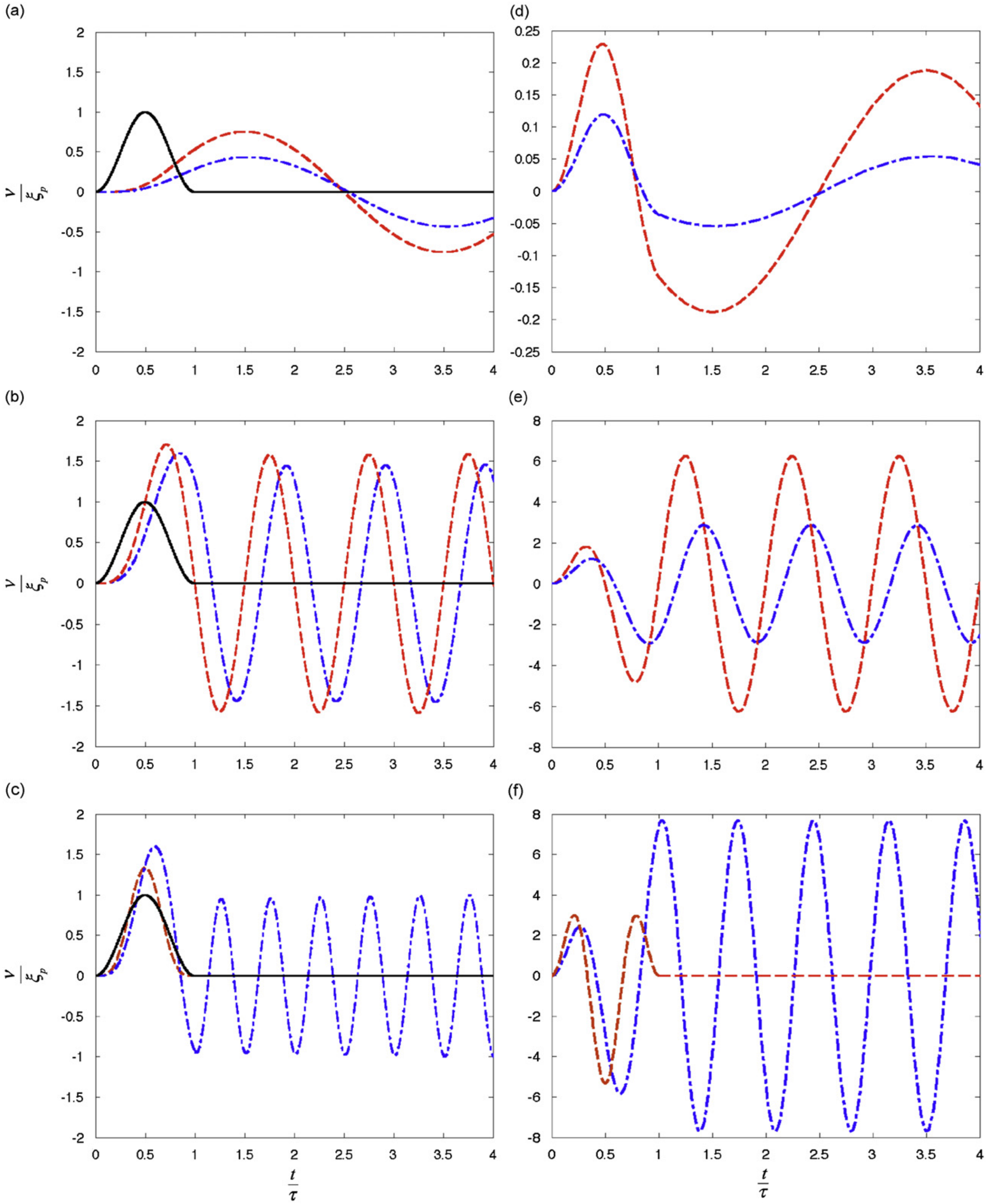

Fig. 5. Normalised time responses for displacement (a)–(c), and normalised accelerations (d)–(f), for a stiffness reduction factor $\Delta k/k = 0.5$: (a) and (d) $\tau/T_1 = 0.25$; (b) and (e) $\tau/T_1 = 1$; (c) and (f) $\tau/T_1 = 2$.(——shock pulse; —passive system; - · -switching system).

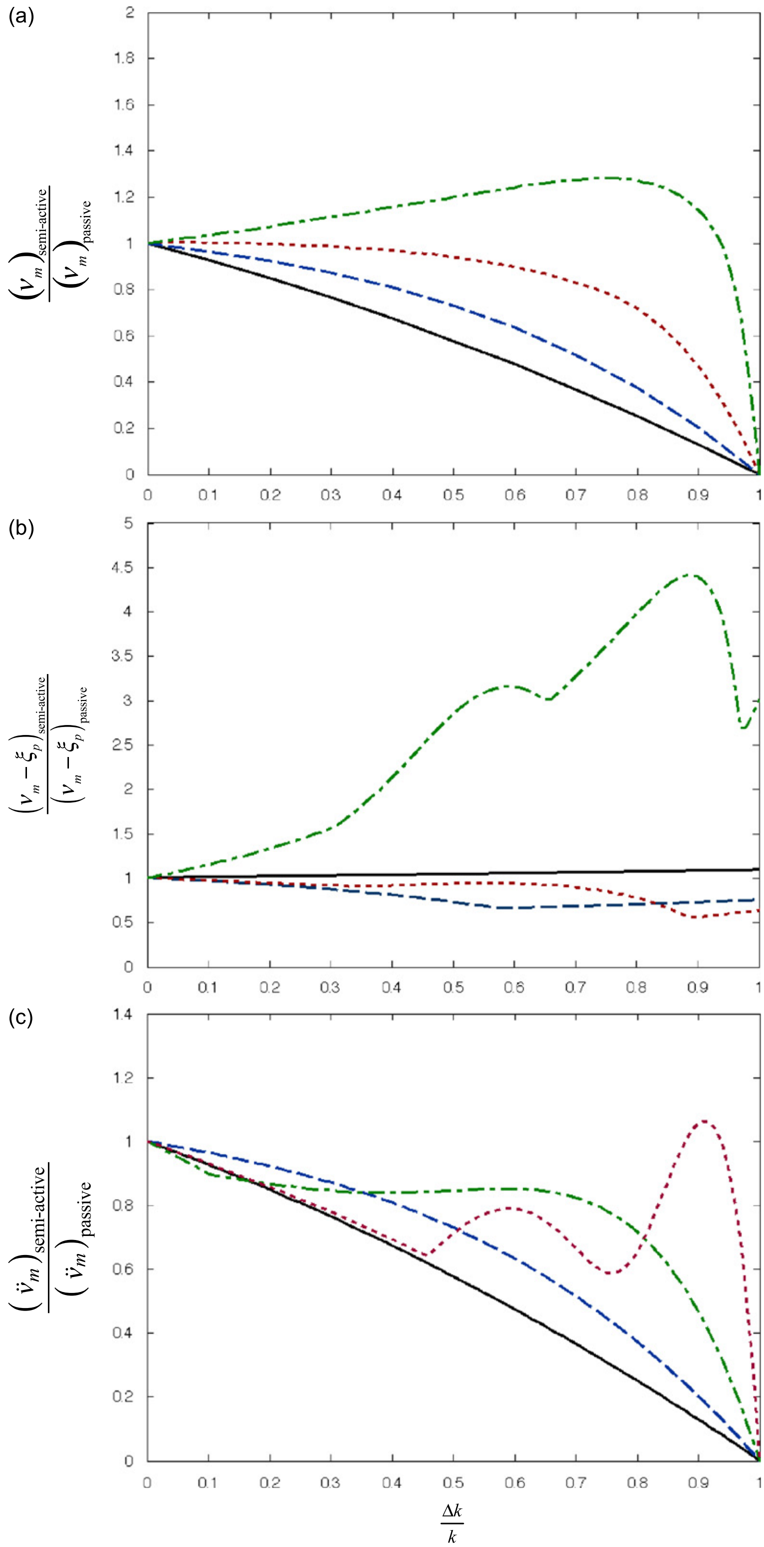

Fig. 6. Ratio between the maximum responses of the switchable stiffness system and the system with the high stiffness state (passive) system, as a function of the ratio of stiffness reduction: (a) absolute displacement, (b) relative displacement, and (c) absolute acceleration. (——τ/T_1=0.25; – – τ/T_1=0.5; …τ/T_1=1; –·– τ/T_1=2).

the acceleration levels. As the initial period ratio increases (i.e. longer pulses or higher initial stiffness) the advantages are less evident. However, the sudden stiffness changes will cause acceleration discontinuities which might be undesirable.

In general, it can be seen that the strategy does not give very good results if $\tau/T_1 > 1$ unless the reduction in stiffness is very high, i.e. larger than 90%. The best results are when there is a significant reduction in the absolute displacement and acceleration, with little or no change observed for the relative motion. This is the case when the original passive system is in the isolation or amplification regions. For low frequency shock isolation, in which the duration of the input is long compared to the natural period, two options may be considered. If the frequency of the shock input is very low, then the system behaves in the quasi-static regime and the isolated mass moves with a displacement and velocity that follows the input. The relative displacement and velocity will typically be small (of the order of 10% of the base amplitude) and the acceleration will also be relatively low because of the low frequency. In this scenario the required stiffness reduction would be impractical and would be problematic if it brings the system into the amplification range of the shock spectra. Fig. 6 shows this issue with the response increasing with any stiffness reduction when the period ratio is higher than 2 for the versed sine input.

In the range where the input excitation is typically of the same frequency as the natural frequency of the mounted mass then much greater benefit is possible. Typically one would aim to achieve a stiffness reduction so that the natural period would be at least twice that of the pulse duration in the case of the versed sine input. Fig. 6 shows that apart from the absolute acceleration there is a reduction compared to the passive system in both the absolute and relative displacement. For high frequency excitation, corresponding to short pulse duration, the passive original isolation has some benefit. If this is not sufficient, then adapting and reducing the suspension stiffness would generally always be beneficial and could be considered.

It is important to note that a delay in switching the stiffness can potentially be important. However, if the delay, which can in practice occur due to several causes such as time lags in the circuitry and sensing configuration, or in the type of actuator used to achieve switchable stiffness, is small compared to the duration of the shock the effect is minimal [16].

4. Control of residual vibrations

4.1. The control strategy

In this section, a strategy is discussed to suppress the residual vibrations that occur after the shock pulse. To illustrate the strategy it is assumed that the mass has an initial velocity and that its displacement is zero. It is the strategy that Chen [12] originally proposed and involves switching the stiffness from a high-state to a low-state and back again during each cycle of vibration. The equation that describes this motion is Eq. (2), but with the right hand side of the equation set to zero, i.e.

$$m\ddot{v}+k_{\text{effective}}v=0 \tag{5}$$

The control strategy has to ensure that the amplitude of vibration decreases every cycle. To achieve this the approach proposed by Onoda et al. [11] is followed. The stiffness should be maximum and equal to k when the product $v\dot{v}$ is positive and minimum and equal to k–Δk when $v\dot{v}$ is negative. This can be expressed mathematically as

$$k_{\text{effective}}=\begin{Bmatrix} k, & v\dot{v}\geq 0 \\ k-\Delta k, & v\dot{v}<0 \end{Bmatrix} \tag{6}$$

When the displacement response satisfies the condition $v\dot{v}\geq 0$ the displacement v and the velocity $\dot{v}$ have the same sign. Applying the control law, the secondary spring, Δk, is disconnected when the absolute value of the displacement of the mass is a maximum. It is connected again when the absolute value of the velocity is maximum, which corresponds to the displacement v being zero and occurs when the system passes through its equilibrium position. It is important to note that it is assumed the unconnected spring has returned to its unstretched position when it is reconnected.

The phase plane plot in Fig. 7(a) shows the solution when the control law is implemented. It demonstrates the effect of switching the isolator stiffness. Additionally, the corresponding displacement as a function of time is shown in Fig. 7(b). The system is piecewise linear between the times that switching occurs and consequently an analytical solution can be found if it is assumed that the spring is reconnected at the static equilibrium position. For the purposes of the simulation, the system has an initial velocity $\dot{v}(0)=a$ and zero initial displacement i.e. $v(0)=0$, which corresponds to the point marked **A** in Figs. 7(a) and (b), and the stiffness is k. When the displacement is maximum, at point **B**, the stiffness Δk is disconnected and the resulting stiffness is k–Δk until point **C**, when the displacement is zero. The stiffness Δk is then reconnected and the total stiffness is equal to k again. It is assumed that the spring Δk has returned to its zero extension position to allow the reconnection to take place. At point **D** the displacement is $v_m\sqrt{1-\Delta k/k}$. The stiffness Δk is disconnected again and reconnected at **E**. The result is a stiffness change every quarter cycle and so the peak amplitude at the nth cycle is related to that at the $(n-1)$th cycle by

$$v_n=v_{n-1}\left(1-\frac{\Delta k}{k}\right) \tag{7}$$

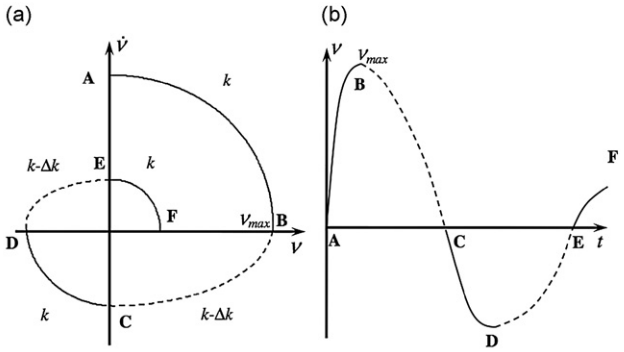

Fig. 7. Effect of the switchable control strategy on the residual vibration: (a) phase plane plot and (b) time history of the displacement response. (—— high stiffness; — low stiffness).

where v_{n-1} is the maximum peak amplitude of the previous cycle. Thus, the amplitude reduces by a factor of $(1-\Delta k/k)$ every cycle. Hence, the vibration can be reduced by having a large stiffness during the first and third quadrants of the phase plane, and a low stiffness in the second and third quadrant. This would cause the trajectory to get closer to the origin every cycle. In this case the orbit would be continuous but not smooth. Its derivatives, particularly the acceleration, would not be continuous and the acceleration would undergo a step change at each stiffness change.

Typical normalised displacement, velocity, and acceleration responses as a function of non-dimensional time were calculated numerically using the Runge–Kutta method and are shown in Fig. 8 for stiffness reductions of 50% and 80%. The time T_a is the average of the two time periods of the unswitched and switched stiffness states T_1 and T_2, i.e. $T_a=(T_1+T_2)/2$. Note that the displacement amplitude decreases at a greater rate for larger values of the stiffness reduction factor $\Delta k/k$. The restoring force on the mass is discontinuous at the points of stiffness reduction (points **B** and **D** in Fig. 7). This condition results in sudden instantaneous acceleration changes of the mass at these points which might be a potential drawback of this system if, for example, the mass is sensitive to acceleration changes. These changes are a result of the sudden decrease in the elastic force acting on the mass when the stiffness Δk disconnects from the mass. These are identifiable by the vertical lines in the graphs of the acceleration, which join the acceleration values during the stiffness switching and could theoretically result in large jerks.

4.2. Energy dissipation mechanism

The semi-active system proposed, which consists of only conservative elements, appears to dissipate energy. In this section an explanation is provided as to why this is possible. The secondary spring Δk is disconnected when its deformation is maximum, i.e. when the potential energy stored in this spring is maximum. It is assumed that the energy in the disconnected spring is dissipated in the interval while it is disconnected. Fig. 9 shows the kinetic, potential, and total energy for a given initial velocity and zero initial displacement. These are normalised with respect to the initial energy in the system. The total energy decreases in steps corresponding to the times when switching occurs. When the secondary spring Δk is disconnected an amount of energy $0.5\Delta k v_{\max}^2$ is removed, where $v_{\max}$ is the displacement at the time of disconnection. Considering that the maximum energy at the beginning of the $(n-1)$th cycle is $0.5kv_{n-1}^2$, and at the nth cycle is $0.5kv_n^2$, the total energy lost during the nth cycle is given using Eq. (7) by

$$\frac{1}{2}kv_{n-1}^2-\frac{1}{2}kv_n^2=\frac{1}{2}kv_{n-1}^2\frac{\Delta k}{k}\left(2-\frac{\Delta k}{k}\right) \tag{8}$$

This energy seems to ‘disappear’ from the system using the present assumptions, which is physically impossible. In practice there are several mechanisms that effectively contribute to this energy dissipation; real springs might possess friction or hysteretic damping. Moreover, in complex systems the energy can be transferred to higher modes of vibration where it is easier to dissipate energy. If the secondary stiffness element is considered massless it will be oscillating at an infinite frequency. The addition of an infinitesimal amount of viscous damping solves this physically unrealisable problem, as the stiffness element returns to the rest position almost immediately, thus dissipating the stored energy in the elastic element. The model with viscous damping is depicted in Fig. 10, where an infinitesimal amount of damping is considered when the secondary stiffness element is disconnected. A further explanation of this phenomenon is given in the appendix.

To relate and compare the decay of the undamped semi-active system with a damped passive system an equivalent viscous damping ratio corresponding to a particular value of the stiffness reduction factor is obtained. The logarithmic decrement δ for a viscously damped sdof system undergoing free vibration is given by [17]

$$\delta=\ln\left(\frac{v_{n-1}}{v_n}\right) \tag{9}$$

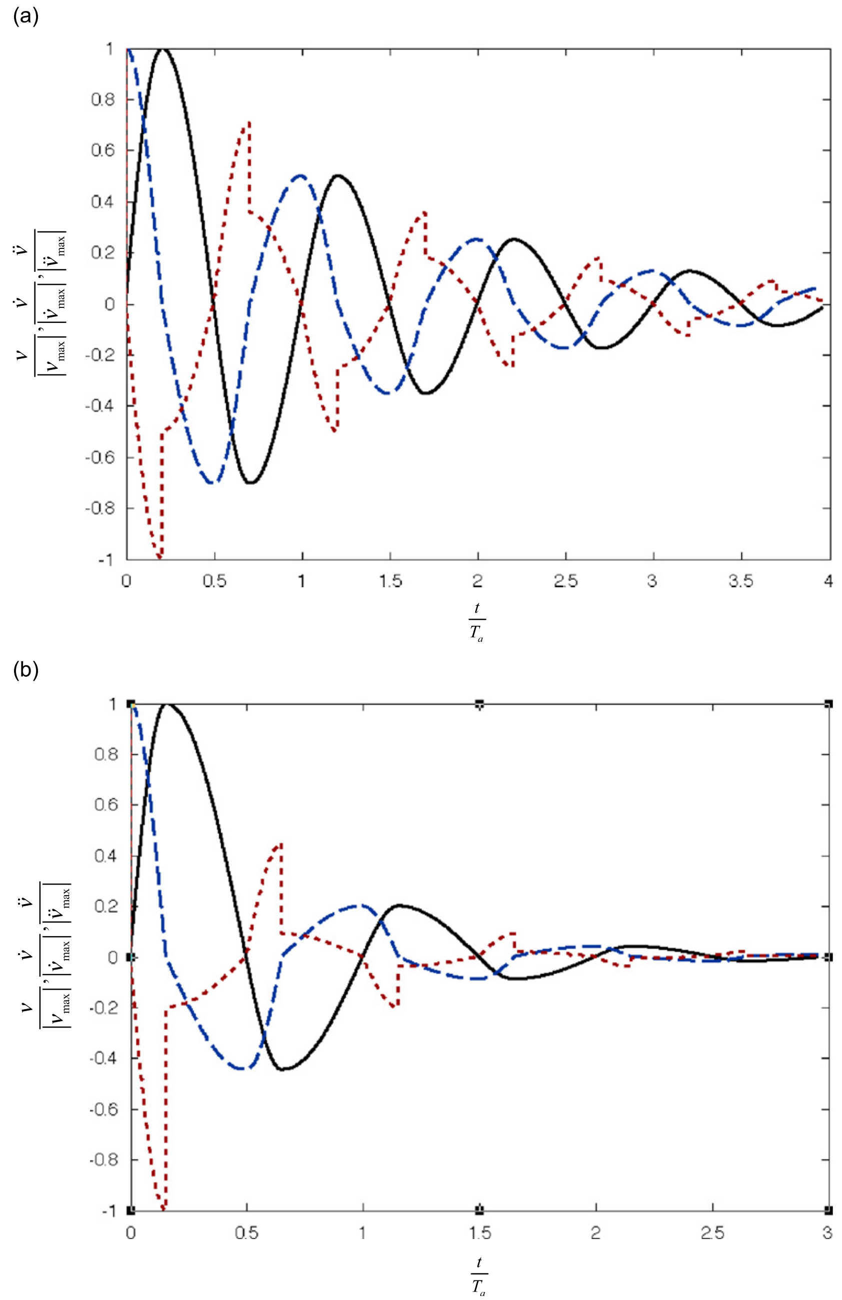

Fig. 8. Typical responses for the residual vibration under semi-active control: (a) $\Delta k/k = 0.5$; (b) $\Delta k/k = 0.8$. (—— displacement; — velocity; …acceleration). T_a is the mean period of the system in the low and high stiffness states. The switching occurring at every quarter cycle of the system.

For the on–off system, the corresponding peak amplitudes are given by Eq. (7). Thus, the equivalent logarithmic decrement for the semi-active system is given by combining Eqs. (7) and (9) to give

$$\delta = -\ln\left(1 - \frac{\Delta k}{k}\right) \tag{10}$$

The equivalent viscous damping ratio is given by [8]

$$\zeta_{eq} = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \tag{11}$$

where δ is given by Eq. (10). Eq. (11) gives the equivalent damping ratio in the sense of the decay per cycle, although the time histories given in Fig. 8 shows that the displacement response is not strictly governed by an exponential decay in time

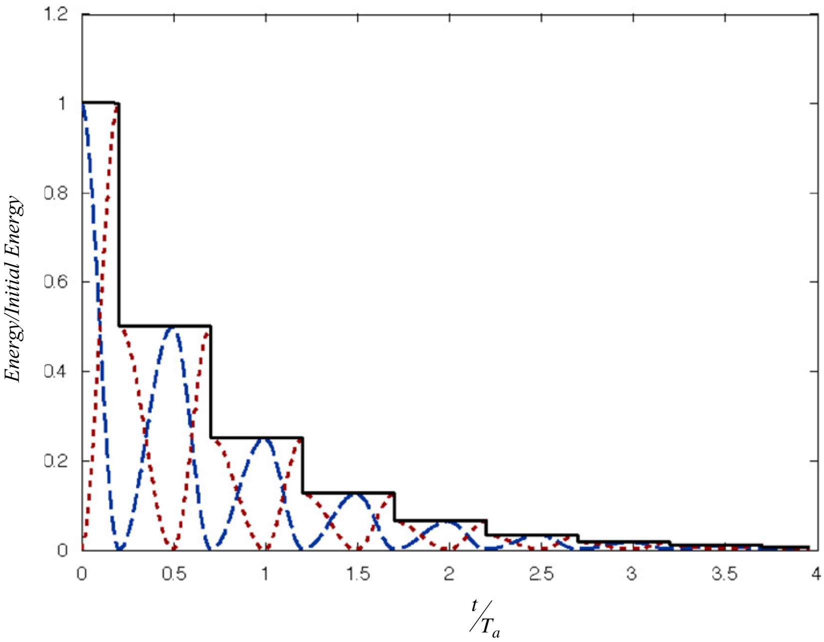

Fig. 9. Example of energy levels for the switchable stiffness system with a stiffness reduction ratio of $\Delta k/k = 0.5$. The horizontal axis represents time normalised with respect to the mean period and vertical axis is the energy normalised with respect to the initial energy (—— total energy; — kinetic energy; …potential energy). T_a is the mean period of the system in the low and high stiffness states. The switching occurring at every quarter cycle of the system.

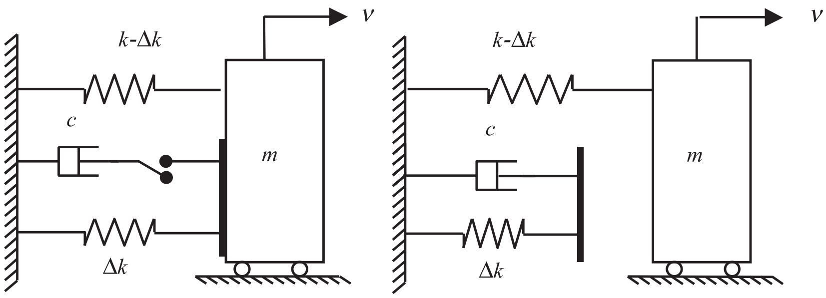

Fig. 10. Switchable on–off model with viscous damping: (a) High stiffness state with both elastic elements connected and the dashpot disconnected. (b) Low stiffness state, the secondary spring has been disconnected and the dashpot is connected to it.

or oscillation at one unique frequency. The equivalent viscous damping ratio ζ_{eq} is plotted in Fig. 11 as a function of the stiffness reduction factor $\Delta k/k$. Also shown is the asymptote $\zeta_{eq} = (\Delta k/k)/2\pi$, which is valid for small values of the stiffness reduction that might occur in practice. In the limit, the stiffness reduction is equal to the total stiffness k and the stiffness ratio $\Delta k/k$ tends to 1. In this case, oscillation no longer occurs and the equivalent viscous damping ratio is unity.

5. Control of shock and the residual vibrations

The two control strategies described above have been combined and the results are compared with a passive system, in which the stiffness has a fixed value. The algorithm to switch the stiffness to control the residual vibrations starts when the shock pulse ends. The secondary spring is assumed to dissipate its stored energy when disconnected (see the appendix for the possible mechanism to ensure that this occurs). To compare with a damped passive system the configuration chosen is as shown in Fig. 12. The viscous damper incorporated into both systems gives a viscous damping ratio $\zeta = c/2\sqrt{km}$ equal to 0.01 for the passive system. It is assumed that damping is always present with the primary stiffness. The secondary stiffness is assumed to lose its stored energy when disconnected, as described earlier. The responses given are for absolute displacement in Fig. 13(a) and acceleration in (b), since the best results are observed in these parameters. The dashed lines represent the passive response for both acceleration and displacement included for comparison. The comparison is between the high stiffness passive state and the switching response considering a stiffness reduction of 50% and a versed

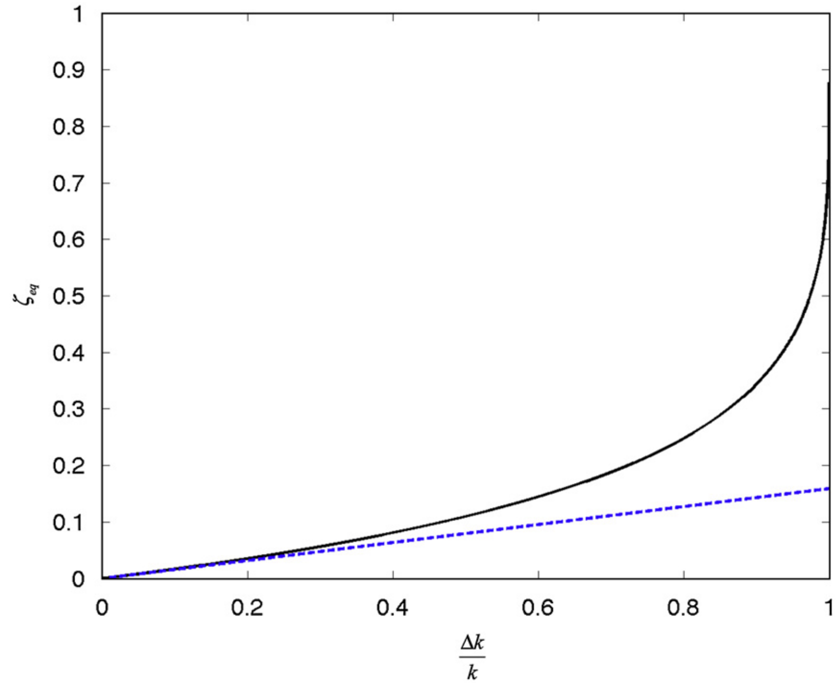

Fig. 11. Equivalent damping ratio as a function of the stiffness reduction ratio. The straight line is the asymptote given by $\zeta_{eq} = (1/2\pi)(\Delta k/k)$ for small values of $\Delta k/k$.

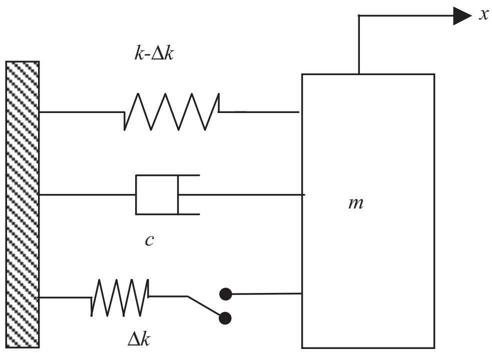

Fig. 12. Schematic of a sdof viscously damped system with switchable stiffness under shock excitation $\xi(t)$ applied to the base.

sine pulse with a period ratio of τ/T_1=0.25. This corresponds to a short duration pulse on a passive system, where the passive system would typically already produce isolation as shown in Fig. 1.

Although some energy is dissipated by the viscous damper included, the viscous damping is small compared to the equivalent damping resulting from the stiffness switching. Nevertheless the damping is considered because all systems have damping to some extent. The benefits of the strategies acting together are easily visible, achieving a reduction in the shock response for the displacement and the acceleration, as well as a faster reduction in the residual vibrations.

6. Conclusions

A theoretical switchable stiffness strategy for shock control has been presented and discussed. The strategy aims to control the shock response in two steps: firstly by reducing the stiffness during a shock pulse, and secondly by switching the stiffness each cycle of vibration during the residual response stage. The strategy has been demonstrated to be capable of reducing the shock response in terms of absolute displacement and acceleration responses, but the benefits on the relative response are small compared with the passive isolation model. Additionally, the residual response can be quickly suppressed by using the switching strategy which can be advantageous for lightly damped systems. An explanation of a potential mechanism for the energy dissipation mechanism has been provided for the suppression of residual vibrations considering an infinitesimal amount of damping in the secondary stiffness element. Finally, both strategies were applied

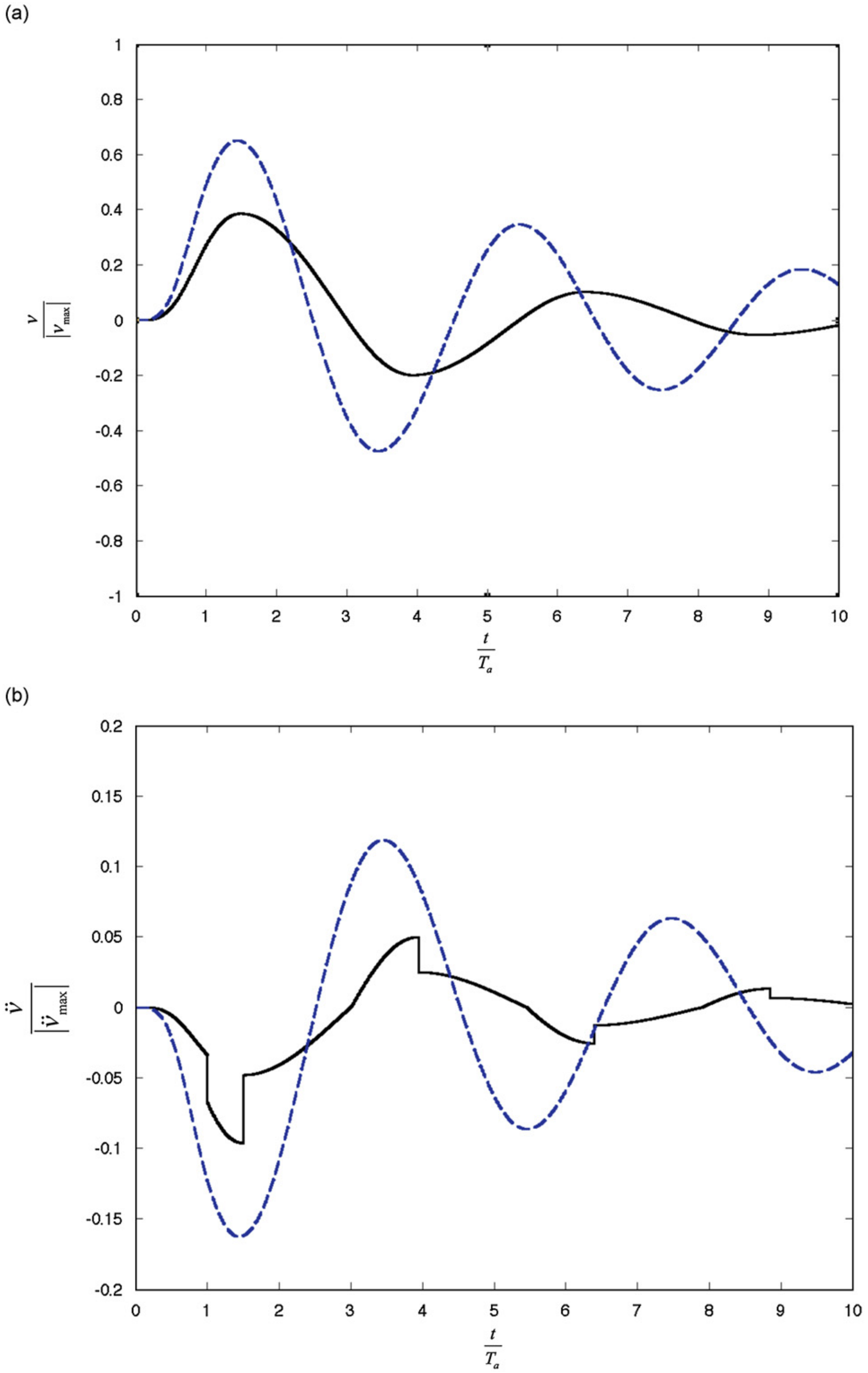

Fig. 13. Passive and switched normalised response, the latter considering both strategies during and after the shock pulse. —— switching system; – passive system: (a) Absolute displacement response and (b) absolute acceleration response. Viscous damping was included with a viscous damping ratio of 0.01. T_a is the mean period of the system in the low and high stiffness states. The switching occurring at every quarter cycle of the system and the stiffness reduction is $\Delta k/k = 0.5$. The versed sine pulse duration to period ratio of the passive system τ/T_1 is equal to 0.25.

together showing the advantages in shock response reduction and residual vibration control showing that the best situation to implement this strategy is in the isolation and amplification regions, as there are possible increases in the response for quasi-static pulses which are long in duration compared to the natural period of the system.

Acknowledgments

The corresponding author would like to acknowledge the support provided by the Mexican Council for Science and Technology (CONACyT) and the Universidad Autonóma de Nuevo Leon, as well as the staff at the Institute of Sound and Vibration Research.

Appendix

Although the model considered is undamped, any real system will include a certain amount of damping. Moreover, if the secondary elastic element is massless, its oscillation frequency tends to infinity. However, if viscous damping is present, even if it is infinitesimal, the secondary system could return to the equilibrium position. The model can then be represented as shown in Fig. 10, where viscous damping has been included. Note that the damping is assumed to be only active when the secondary system is disconnected, as the switch in Fig. 10(a) indicates. As a result, the secondary system becomes a first-order system, whose response is given by [15]

$$v = v(0)e^{-(\Delta k/c)t} \tag{A.1}$$

where $v(0)$ is the displacement of the secondary system at the time of disconnection corresponding to t=0. An important point is to ensure that the secondary spring Δk returns to its equilibrium position, and this depends upon the amount of damping present. An upper limit for the permissible damping for the secondary spring–damper system can be calculated considering the time while it is disconnected, which is a quarter of the natural period for the main system in its low stiffness state. Considering that the natural frequency when the system is on its low stiffness state is $\omega_{\text{low}} = \sqrt{k-\Delta k/m}$ and that the period is $2\pi/\omega_{\text{low}}$ in its low stiffness state then a quarter of the period can be written as

$$t_0 = \frac{\pi}{2\sqrt{(k-\Delta k)/m}} \tag{A.2}$$

As the time available for the secondary system to get back to equilibrium depends upon the factor $\Delta k/k$ this will also determine the permissible amount of damping in the system. It is important to recall that in a massless first-order system more damping will cause a longer time to get back to rest [17]. Rearranging Eq. (A.1) for c and substituting the settling time to t_0, one can write

$$c = \frac{-\Delta k t_0}{\ln(v_{\text{final}}/v(0))} \tag{A.3}$$

v_{final} is the specified or required final displacement of the secondary spring–damper system which must be as close to zero as possible. The latter expression can be written in non-dimensional form as an equivalent viscous damping ratio

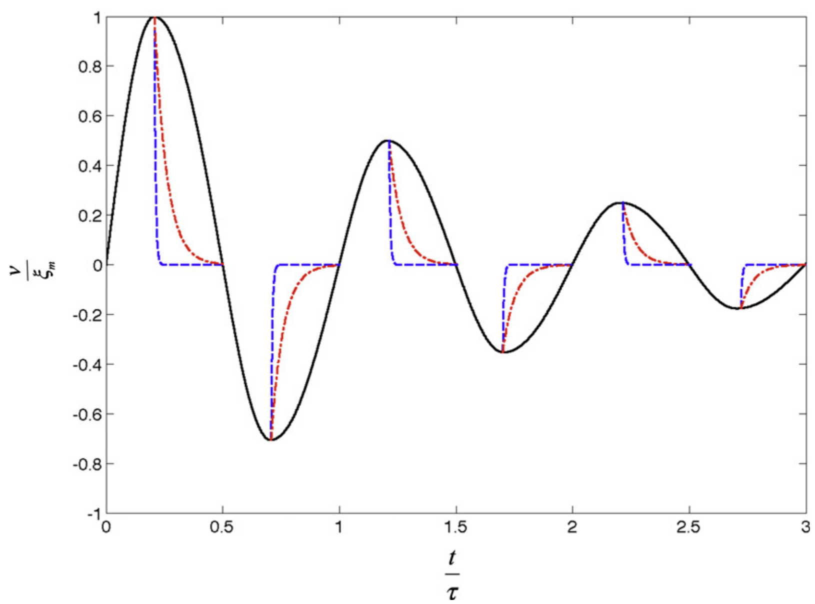

Fig. A1. Response of the combined main system and the secondary spring–damper system for different damping values considering a stiffness reduction ratio of s=0.5. (— main system response; – – $c/2\sqrt{km} = 0.1$; –·– $c/2\sqrt{km} = 0.01$).

for a linear system:

$$\frac{c}{2\sqrt{km}} = \frac{-(\Delta k/k)\pi}{2(1-\Delta k/k)}\frac{1}{\ln(v_{\text{final}}/v_0)} \tag{A.4}$$

where $\Delta k/k$ is the stiffness reduction factor, strictly less than 1.

The corresponding time histories for both the main and the secondary systems are shown in Fig. A1. For this figure it was considered a final displacement v_{final} to be 0.1% of the initial displacement at the time of disconnection. It can be observed that the secondary system, comprising the stiffness element Δk and the dashpot c, returns very closely to the equilibrium position while disconnected and then reconnects again when the mass is at its static equilibrium position. Note that the smaller the damping the less time it takes to return to rest. If the damping is negligible, the model further reduces to the simplest model described at the beginning of the section, but the secondary stiffness element will be oscillating at an infinite frequency because it is assumed to have zero mass. The addition of an infinitesimal amount of viscous damping solves this physically unrealisable problem, as the stiffness element returns to the equilibrium position, thus dissipating the stored energy in the elastic element.

References

[1] C.M. Harris, C.E. Crede, *Shock and Vibration Handbook*, McGraw-Hill, New York, 1996.
[2] R.L. Eshleman, P. Rao, Response of mechanical shock isolation elements to high rate input loading, *Shock and Vibration Bulletin* 40 (5) (1969) 217–234 Shock and Vibration Information Center.
[3] R.S. Ayre, *Engineering Vibrations*, McGraw-Hill, New York, 1958.
[4] J.C. Snowdon, Response of nonlinear shock mountings to transient foundation displacements, *Journal of the Acoustical Society of America* 33 (10) (1961) 1295–1304.
[5] J.C. Snowdon, Transient response of nonlinear isolation mountings to pulse-like displacements, *Journal of the Acoustical Society of America* 35 (3) (1963) 389–396.
[6] D.V. Balandin, N.N. Bolotnik, W.D. Pilkey, Review: optimal shock and vibration isolation, *Shock and Vibration* 5 (2) (1998) 73–87.
[7] D.V. Balandin, N.N Bolotnik, W.D. Pilkey, Pre-acting control for shock and impact isolation systems, *Shock and Vibration* 12 (1) (2005) 49–65.
[8] M.F. Winthrop, W.P. Baker, R.G. Cobb, A variable stiffness device selection and design tool for lightly damped structures, *Journal of Sound and Vibration* 287 (4–5) (2005) 667–682.
[9] J. Onoda, K. Minesugi, Alternative control logic for type-II variable-stiffness system, *AIAA Journal* 34 (1) (1996) 207–209.
[10] J. Onoda, T. Endo, H. Tamaoki, N. Watanabe, Vibration suppression by variable-stiffness members, *AIAA Journal* 29 (6) (1991) 977–983.
[11] J. Onoda, T. Sano, K. Kamiyama, Active, passive, and semi-active vibration suppression by stiffness variation, *AIAA Journal* 30 (12) (1992) 2922–2929.
[12] J.C. Chen, Response of large spacecraft structures with stiffness control, *Journal of Spacecrafts and Rockets* 21 (5) (1984) 163–167.
[13] F Jabbari, J.E. Bobrow, Vibration suppression with a resettable device, *Journal of Engineering Mechanics* 128 (9) (2002) 916–924.
[14] J.L. Leavitt, F. Jabbari, J.E. Bobrow, Optimal performance of variable stiffness devices for structural control, *Journal of Dynamic Systems, Measurement and Control—Transactions of the ASME* 129 (2) (2007) 171–177.
[15] N. Zhou, K. Liu, A tunable high-static-low-dynamic stiffness vibration isolator, *Journal of Sound and Vibration* 329 (9) (2010) 1254–1273.
[16] D.F. Ledezma-Ramirez, Shock Isolation using Switchable Stiffness, PhD Thesis, University of Southampton, 2008.
[17] W.J. Palm, *Mechanical Vibrations*, John Wiley and Sons, New York, 1990.